The Neutrino – Massless or Massive?

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Pauli’s Letter

4th of December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the “wrong” statistics of the N and $^6$Li nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the “exchange theorem” of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...
Pauli’s Letter

I agree that my remedy could seem incredible because one should have seen these neutrons much earlier if they really exist. But only the one who dare can win and the difficult situation, due to the continuous structure of the beta spectrum, is lighted by a remark of my honoured predecessor, Mr Debye, who told me recently in Bruxelles: “Oh, It’s well better not to think about this at all, like new taxes”. From now on, every solution to the issue must be discussed. Thus, dear radioactive people, look and judge.

Unfortunately, I cannot appear in Tubingen personally since I am indispensable here in Zurich because of a ball on the night of 6/7 December. With my best regards to you, and also to Mr Back.

Your humble servant,

W. Pauli
Pauli’s Letter

Wolfgang Ernst Pauli (April 25, 1900 – December 15, 1958)

Austrian physicist who postulated the “neutrino”.

Austrian physicist who postulated the “neutrino”.
Neutrinos

Neutrinos are:

- elementary particles (s.c. leptons – same family as the electron)
- divided in (at least) three different s.c. flavors
- electrically uncharged (neutral) and interacting only via the weak force
- nearly massless (i.e. they have very small masses)
- very elusive (i.e. they have very small cross-sections)

But:
The neutrino is the most frequent particle in the Universe.
Different Neutrinos and Charged Leptons

<table>
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<tr>
<th>Family</th>
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</tr>
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<td>$\nu_\mu$</td>
<td>$\nu_\tau$</td>
</tr>
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<table>
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<tr>
<th>Neutrino</th>
<th>Year discovered</th>
<th>Discovered by</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\nu}_e$</td>
<td>1956</td>
<td>C.L. Cowan, Jr. &amp; F. Reines</td>
</tr>
<tr>
<td>$\nu_\mu$</td>
<td>1962</td>
<td>L.M. Lederman, M. Schwartz &amp; J. Steinberger</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>2000</td>
<td>DONUT collaboration at Fermilab, USA</td>
</tr>
</tbody>
</table>
Neutrino Sources

- the Sun ($\sim 2 \cdot 10^{38}$ neutrinos per second)
- the atmosphere (from cosmic rays)
- accelerators
- nuclear power plants ($\sim 10^{20}$ neutrinos per second)
- "natural radioactivity"
- supernovae explosions ($\sim 6 \cdot 10^{57}$ neutrinos per second)
- "Big Bang", cosmic point-sources
- etc.
Solar Neutrinos

The Sun:

The Standard Solar Model (SSM) predicts that the Sun shines because of fusion reactions in the core of the Sun.

SSM: $\sim 2 \cdot 10^{38}$ neutrinos per second

The distance between the Sun and the Earth:

$\sim 1.5 \cdot 10^{11}$ m
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$$\Phi_\nu = \frac{N_\nu}{4\pi R^2} \sim \frac{2 \cdot 10^{38}}{4\pi (1.5 \cdot 10^{13})^2} \text{cm}^{-2}\text{s}^{-1} \sim 7 \cdot 10^{10} \text{cm}^{-2}\text{s}^{-1}$$
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\]

\( \therefore \) Ca 70 billion solar neutrinos passes through your thumbnail of ca 1 cm\(^2\) per second!
How Many Different Types of Neutrinos Are There?

Experimental studies of $Z^0$-decays at LEP:

$$N_\nu = 2.984 \pm 0.008$$

⇒ There are three neutrino flavors.

---

How Many Different Types of Neutrinos Are There?

The number of different neutrinos from astrophysical data:

(S. Hannestad, 2003)

1. WMAP + Wang + 2dFGRS + HST + SN-Ia:
   \[ N_\nu = 4.0^{+3.0}_{-2.1} \text{ (at } 95\% \text{ C.L.)} \]

   + BBN \( ^4\text{He och D} \): \[ N_\nu = 2.6^{+0.4}_{-0.3} \text{ (at } 95\% \text{ C.L.)} \]

2. WMAP + 2dFGRS: \[ N_\nu = 3.1^{+3.9}_{-2.8} \]

3. WMAP: \[ N_\nu = 2.1^{+6.7}_{-2.2} \]

4. Hannestad & Raffelt, 2006: \[ 2.7 < N_\nu < 4.6 \text{ (at } 95\% \text{ C.L.)} \]
How Many Different Types of Neutrinos Are There?

The number of different neutrinos from "Big Bang" nucleosynthesis:

(S. Burles, K.M. Nollett och M.S. Turner, 1999)

⇒ There are three neutrino flavors.
Neutrino Oscillations

Quantum mechanics:
Neutrinos are massive and mixed $\Leftrightarrow$ Neutrino oscillations

**Neutrino mixing:**

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}.
$$

The leptonic mixing matrix:

$$
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
0 & e^{i\delta_{CP}} s_{13}
\end{pmatrix}
0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0
\end{pmatrix}
0 & 0 & 1
\end{pmatrix},
$$

where $c_{ab} \equiv \cos \theta_{ab}$ and $s_{ab} \equiv \sin \theta_{ab}$.

"atmospheric neutrinos" "reactor neutrinos" "solar neutrinos"
Neutrino Oscillations

A graphical representation of neutrino mixing:
Neutrino Oscillations

The Pontecorvo formula:

\[ P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E_\nu} \frac{L}{amplitude} \]

where \( \theta \) is the mixing angle, \( \Delta m^2 \) is the mass squared difference, \( E_\nu \) is the neutrino energy, and \( L \) is the baseline length.

It holds that:

\[ P(\nu_e \rightarrow \nu_e) = 1 - P(\nu_e \rightarrow \nu_\mu) = 1 - P(\nu_\mu \rightarrow \nu_e) = P(\nu_\mu \rightarrow \nu_\mu) \]
Neutrino Oscillations

A graphical representation of neutrino oscillations in a two-flavor approximation:
Neutrino Oscillation Experiments

Experiment: Super-Kamiokande

Measurements of atmospheric neutrinos and solar neutrinos.

Reaction: Elastic scattering \( \nu_\alpha + e^- \rightarrow \nu_\alpha + e^- \), \( \alpha = e, \mu, \tau \)

June 1998: Strong evidence for that neutrino oscillations occur and that neutrinos are massive.
Neutrino Oscillation Experiments

Atmospheric neutrinos – Super-Kamiokande (1289 days)

Neutrino fluxes

"Up-going":
\[
\cos \theta = -1 \iff \theta = 180^\circ
\]

"Down-going":
\[
\cos \theta = 1 \iff \theta = 0
\]

(T. Toshito, 2001)
Neutrino Oscillation Experiments

Solar neutrinos – Super-Kamiokande (1258 days)

The angular distribution of solar neutrino candidates


The Sun in "neutrino light"
Neutrino Oscillation Experiments


CC: \( \nu_e + d \to e^- + p + p, \quad E_{\text{th}} \simeq 1.44 \text{ MeV}, \)

NC: \( \nu_\alpha + d \to \nu_\alpha + p + n, \quad \alpha = e, \mu, \tau, \quad E_{\text{th}} \simeq 2.23 \text{ MeV}, \)

ES: \( \nu_\alpha + e^- \to \nu_\alpha + e^-, \quad \alpha = e, \mu, \tau. \)

\[
\text{CC} \quad \Rightarrow \quad \phi_{\nu_e} \simeq 1.76 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1} \\
\text{NC} \quad \Rightarrow \quad \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau} \simeq 5.09 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1} \\
\text{ES} \quad \Rightarrow \quad \phi_{\nu_e} + \frac{1}{6} (\phi_{\nu_\mu} + \phi_{\nu_\tau}) \simeq 2.39 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1} \\
\Rightarrow \quad \phi_{\text{SNO}} \simeq 5.09 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1} \quad [\phi_{\text{SSM}} \simeq 5.05 \cdot 10^6 \text{ cm}^{-2}\text{s}^{-1}]
Mixing Angles and Mass Squared Differences

Definition (leptonic mixing and mixing angles):

\[
\begin{pmatrix}
C_{13}C_{12} & S_{12}C_{13} & S_{13}e^{-i\delta_{CP}} \\
-S_{12}C_{23} - S_{23}S_{13}C_{12}e^{i\delta_{CP}} & C_{23}C_{12} - S_{23}S_{13}S_{12}e^{i\delta_{CP}} & S_{23}C_{13} \\
S_{23}S_{12} - S_{13}C_{23}C_{12}e^{i\delta_{CP}} & -S_{23}C_{12} - S_{13}S_{12}C_{23}e^{i\delta_{CP}} & C_{23}C_{13}
\end{pmatrix}
\]

where \(S_{ab} \equiv \sin \theta_{ab}\) and \(C_{ab} \equiv \cos \theta_{ab}\).

Definition (neutrino mass squared differences):

\[
\Delta m^2_{ab} = m_a^2 - m_b^2,
\]

where \(m_a\) \((a = 1, 2, 3)\) is the mass for the \(a\)th neutrino mass eigenstate.

Mixing angles and mass squared differences are fundamental neutrino parameters in neutrino oscillation experiments.
Mixing Angles and Mass Squared Differences

Current values (June 30, 2006) of the fundamental three-flavor neutrino oscillation parameters [including data from the Sun, the atmosphere, reactors (KamLAND & CHOOZ), and accelerators (K2K & MINOS)]:

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<th>Parameter</th>
<th>Best-fit value</th>
<th>Interval ((3\sigma))</th>
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<td>(\Delta m_{21}^2)</td>
<td>(7.9 \cdot 10^{-5} \text{ eV}^2)</td>
<td>((7.1 \div 8.9) \cdot 10^{-5} \text{ eV}^2)</td>
</tr>
<tr>
<td>(\Delta m_{31}^2)</td>
<td>(2.6 \cdot 10^{-3} \text{ eV}^2)</td>
<td>((2.0 \div 3.2) \cdot 10^{-3} \text{ eV}^2)</td>
</tr>
<tr>
<td>(\theta_{12})</td>
<td>33.2°</td>
<td>29.3° (\div) 39.2°</td>
</tr>
<tr>
<td>(\theta_{13})</td>
<td>-</td>
<td>0 (\div) 11.5°</td>
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<td>(\theta_{23})</td>
<td>45.0°</td>
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<td>$[0, 2\pi)$</td>
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$\therefore$ Nearly bimaximal leptonic mixing, i.e., $θ_{12}$ and $θ_{23}$ are large and $θ_{13}$ is small.
Comparison: Quarks and Leptons

Quark mixing can be represented by a similar mixing matrix as the one for leptonic mixing. This mixing matrix is usually called the CKM matrix. The values of the mixing angles are:

\[ \theta_{12}^{\text{CKM}} = 13.0^\circ \pm 0.1^\circ \quad \theta_{13}^{\text{CKM}} = 0.2^\circ \pm 0.1^\circ \quad \theta_{23}^{\text{CKM}} = 2.4^\circ \pm 0.1^\circ \]
\[ \delta_{\text{CP}}^{\text{CKM}} = 1.05 \pm 0.24 \]
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\]

The leptonic mixing matrix is sometimes called the MNS or PMNS matrix. The values of the mixing angles are:

\[
\begin{align*}
\theta_{12}^{\text{MNS}} & \approx 332^\circ & \theta_{13}^{\text{MNS}} & \lesssim 11.5^\circ & \theta_{23}^{\text{MNS}} & \approx 45.0^\circ & \delta_{\text{CP}}^{\text{MNS}} & = ?
\end{align*}
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\theta_{23}^{\text{MNS}} &\approx 45.0^\circ \\
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\end{align*}
\]

Quarks (CKM): 3 small mixing angles
Leptons (MNS): 2 large mixing angles and 1 small mixing angle
Open Questions Concerning the Neutrino Parameters

The standard model (SM) for particle physics, which assumes that neutrinos are massless, have 18 free parameters. The neutrino parameters are six. Thus, the SM for massive neutrinos have 24 free parameters.

- What is the sign of $\Delta m^2_{31}$? Do we have normal mass hierarchy, i.e. $m_1 < m_2 < m_3$? Or do we have inverted mass hierarchy, i.e. $m_3 < m_1 < m_2$?

- What is the upper bound on the mixing angle $\theta_{13}$? There is ongoing research in order to determine or at least make the upper bound better for the mixing angle $\theta_{13}$.

- What is the value of the CP-violating phase $\delta_{\text{CP}}$? This parameter is so far totally undetermined!
The Absolute Neutrino Mass Scale

What are the masses of the neutrino mass eigenstates, i.e. what are the values of $m_1$, $m_2$, and $m_3$?
∴ 3 masses
The Absolute Neutrino Mass Scale

What are the masses of the neutrino mass eigenstates, i.e. what are the values of $m_1$, $m_2$, and $m_3$?
\[ \therefore 3 \text{ masses} \]

From neutrino oscillations and neutrino oscillation experiments, one can only determine the mass squared differences $\Delta m_{31}^2$ and $\Delta m_{21}^2$.
In addition, one does not know the sign of $\Delta m_{31}^2$.
\[ \therefore 2 \text{ mass squared differences} \]
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From neutrino oscillations and neutrino oscillation experiments, one can only determine the mass squared differences $\Delta m_{31}^2$ and $\Delta m_{21}^2$. In addition, one does not know the sign of $\Delta m_{31}^2$. ∴ 2 mass squared differences

Thus, one cannot determine the absolute neutrino mass scale via neutrino oscillation experiments!
Neutrino Masses

Assume that the neutrino mass eigenstates $\nu_1$, $\nu_2$, and $\nu_3$ are the primary components of the neutrino flavor eigenstates $\nu_e$, $\nu_\mu$, and $\nu_\tau$. Then, the present upper bound on the neutrino masses are:

- The electron neutrino: $m_{\nu_e} < 3 \text{ eV} \quad (^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e)$
- The muon neutrino: $m_{\nu_\mu} < 0.19 \text{ MeV} \quad (\pi^\pm \rightarrow \mu^\pm + \nu_\mu)$
- The tau neutrino: $m_{\nu_\tau} < 18.2 \text{ MeV} \quad (\tau \rightarrow 5\pi + \nu_\tau)$

But:

One should instead determine the upper bounds on the masses of the neutrino mass eigenstates, since they are the only states that have well-defined masses.
Neutrino Masses

The sum of neutrino masses:

\[ M_\nu \equiv \sum_a m_a , \]

where \( m_a \) is the mass for the \( a \)th neutrino mass eigenstate.
Neutrino Masses

Some upper bounds on the sum of neutrino masses $M_\nu$ from cosmological data (@ 95 % C.L.):

<table>
<thead>
<tr>
<th>Authors</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elgarøy et al. (2002)</td>
<td>1.8 eV</td>
</tr>
<tr>
<td>Sanchez et al. (2005)</td>
<td>1.2 eV</td>
</tr>
<tr>
<td>Goobar et al. (2006)</td>
<td>0.62 eV</td>
</tr>
<tr>
<td>Fukugita et al. (2006)</td>
<td>2.0 eV</td>
</tr>
<tr>
<td>Spergel et al. (2006)</td>
<td>0.68 eV</td>
</tr>
<tr>
<td>Seljak et al. (2006)</td>
<td>0.17 eV</td>
</tr>
</tbody>
</table>

(Ø. Elgarøy and O. Lahav, 2006)

$\Rightarrow M_\nu \lesssim 1 \text{ eV}$
Determination of the Masses?

If one assumes a normal mass hierarchy, i.e. $m_1 < m_2 < m_3$, and uses the measured values of the mass squared differences from neutrino oscillation experiments and the cosmological upper bound on the sum of neutrino masses. Can one then determine the neutrino masses?
Determination of the Masses?

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Solve the non-linear system of equations:

\[
\begin{align*}
    m_3^2 - m_1^2 &= +|\Delta m_{31}^2| = +2.6 \cdot 10^{-3} \text{ eV}^2 \\
    m_2^2 - m_1^2 &= \Delta m_{21}^2 = 7.9 \cdot 10^{-5} \text{ eV}^2 \\
    m_1 + m_2 + m_3 &= M_\nu = (0.1 \div 1) \text{ eV}
\end{align*}
\]
Determination of the Masses?

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$$\begin{cases} m_3^2 - m_1^2 = +|\Delta m_{31}^2| = +2.6 \cdot 10^{-3} \text{ eV}^2 \\ m_2^2 - m_1^2 = \Delta m_{21}^2 = 7.9 \cdot 10^{-5} \text{ eV}^2 \\ m_1 + m_2 + m_3 = M_\nu = (0.1 \div 1) \text{ eV} \end{cases}$$

This system of equations has a valid solution!

$$m_1 \sim (0.021 \div 0.33) \text{ eV} \quad m_2 \sim (0.023 \div 0.33) \text{ eV} \quad m_3 \sim (0.055 \div 0.34) \text{ eV}$$
Summary

✔ Neutrinos are very elusive particles, but are the most frequent particles in the Universe. 
\[ B : \gamma : \nu \sim 1 : 10^9 : 10^{10} \]

✔ Neutrinos are massive and mixed. \( \Leftrightarrow \) Oscillations occur among different flavors.

✔ There are most probably three neutrino flavors.

✔ The absolute neutrino mass scale is not known.

✔ Three of the six neutrino parameters need to be better determined.

✔ The sum of neutrino masses (for the neutrino mass eigenstates) is \( \mathcal{O}(1) \text{ eV} \).